

Here we practice a few problems on conditional probability.

To solve conditional probability, the most common way is to spell out the formula based on the definition

$$P(A|B) = P(A \text{ and } B) / P(B)$$

1) Roll a fair, six-sided die and recording the number that lands face up. $S = \{1, 2, 3, 4, 5, 6\}$. Consider the following event:

$A = \{1\}$, $B = \{1, 3, 5\}$, Find $P(A|B)$, the probability of rolling a one given that an odd number was rolled.

2) According to US census, for married-couple family, 84.9% of all fathers are employed; and 57.5% of all these family have both parents employed. Suppose a married family is randomly selected, given that the father is employed, what is the probability that the mother is employed?

3) Do a conditional probability calculation on a two-way contingency table.

Next we study on Section 4.5 about independence, multiplication rule and Bayes rule.

If A and B are independent, two ways of verifying it:

1) $P(A|B) = P(A)$, or $P(B|A) = P(B)$.

2) $P(A \cap B) = P(A)P(B)$.

The multiplication rule for independent events:

If A_1, A_2, \dots, A_n are independent, $P(A_1 \cap \dots \cap A_n) = P(A_1) \times \dots \times P(A_n)$.

On the other hand, if the events are not dependent, we also have a multiplication rule:

$$P(A \cap B) = P(A)P(B|A).$$

$$P(A \cap B \cap C) = P(A)P(B|A)P(C|A \cap B).$$

Examples:

1) Suppose 99.4% mattress deliveries are on time. Two deliveries are selected. a) What is the probability that both are on time? b) What is the probability that both are late? What is the probability that exactly one is on time?

2) Sales agents contacted people by phone to sell some product. If they contact a female customer, the probability of making a deal is 0.65; if they contact a male customer, the probability of making a deal is 0.55. If 50% of the population is male, what is the probability of making a deal with a male customer?

Next let us look at the interesting famous Monty Hall problem.