Today we will add the conditional probability to the tree diagram.

Example one:

If in a population, the pencentage of individuals carrying HIV virus is 0.5%. The HIV blood test has a 5% false positive rate (giving positive result to healthy individual) and 10% false negative rate (giveing negative result to virus carrier). Suppose a random chosen individual has taken the test and what is the probability of him receiving positive result?

The example above follows the natural prospetive thinking: know the probability that A (first stage experiment) occurs, and then know the probability of B (second stage experiment) given A and given A^c, last we infer the probabily of B (second stage).

Sometimes our thinking follows a retrospective view: Given B occurs (second stage), we wish to infer what is the probability that A actually occured instead of A^c in the first stage?

Example one continued:

Suppose this individual got a positive result and he is in panic. He went to his family physician and his family doctor calculated the conditional probality that he is a carrier given that he has a positive result? The family doctor suggested him to repeat the test, what is the conditional probability that he is a carrier given that he has two positive results?

To summarize the approach above, we have the following fact and theorem.

Fact: If the sample space is decomposed into k non-overlapping subsets: $S = \cup_{i=1}^{k} A_k$. Then for any event B, $P(B) = \sum_{i=1}^{k} P(B \cap A_k)$.

Bayes Theorem: If the sample space is decomposed into k non-overlapping subsets: $S = \cup_{i=1}^{k} A_k$. Then the conditional probability

$$P(A_k|B) = \frac{P(A_k \cap B)}{P(B)} = \frac{P(A_k \cap B)}{\sum_{i=1}^{k} P(A_k \cap B)}.$$

Now we finish chapter 4. We conclude with a few more challenging questions.

1) You toss a fair coin three times:

What is the probability of three heads, HHH? What is the probability that you observe exactly one heads? Given that you have observed at least one heads, what is the probability that you observe at least two heads?

2)Problem For three events AA, BB, and CC, we know that

A and C are independent, B and C are independent, A and B are disjoint, $P(A \cup C) = 2/3$, $P(B \cup C) = 3/4$, $P(A \cup B \cup C) = 11/12$, Find P(A),P(B)and P(C).

3)In my town, it's rainy one third of the days. Given that it is rainy, there will be heavy traffic with probability 1/2, and given that it is not rainy, there will be heavy traffic with probability 1/4. If it's rainy and there is heavy traffic, I arrive late for work with probability 1/2. On the other hand, the probability of being late is reduced to 1/8 if it is not rainy and there is no heavy traffic. In other situations (rainy and no traffic, not rainy and traffic) the probability of being late is 0.25. You pick a random day.

What is the probability that it's not raining and there is heavy traffic and I am not late? What is the probability that I am late? Given that I arrived late at work, what is the probability that it rained that day?

Now we start our discussion on Chapter 5.